ECE 536 – Spring 2022

Homework #6 – Solutions

Problem 1) project proposal (no solution)
Problem 2)

(a) We have the following parameters:

Mirror Reflectivities $R_1 = R_2 = R = 0.3$
Cavity Length $L = 600\ \mu\text{m} = 600 \times 10^{-4}$ cm
Loss in the waveguide core (active region) $\alpha_g = 5\ \text{cm}^{-1}$
Loss in the waveguide cladding $\alpha_0 = 13\ \text{cm}^{-1}$
Confinement factor $\Gamma = 0.9$

Review the various relations for the laser:

Net gain in the active region (guide core) = Loss in the cladding + Mirrors Loss

$$\Gamma\left(g_{th} - \alpha_g\right) = (1-\Gamma)\alpha_0 + \alpha_m$$

Intrinsic Total Loss of the cavity = Loss in the guide core + Loss in the cladding

$$\alpha_i = \alpha_g \Gamma + (1-\Gamma)\alpha_0$$

Mirror Loss

$$\alpha_m = \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right) = \frac{1}{2L}\ln\left(\frac{1}{R^2}\right) = \frac{1}{L}\ln\left(\frac{1}{R}\right)$$

Threshold gain condition: modal gain equal total loss

$$\Gamma g_{th} = \alpha_i + \alpha_m$$

Gain at threshold

$$g_{th} = \frac{\alpha_i + \alpha_m}{\Gamma}$$

We have:

$$\alpha_i = \alpha_g \Gamma + (1-\Gamma)\alpha_0 = 5\times 0.9 + 0.1\times 13 = 5.8\ \text{cm}^{-1}$$

$$\alpha_m = \frac{1}{L}\ln\left(\frac{1}{R}\right) = \frac{1}{600\times 10^{-4}}\ln\left(\frac{1}{0.3}\right) = 20.0662\ \text{cm}^{-1}$$

$$g_{th} = \frac{\alpha_i + \alpha_m}{\Gamma} = \frac{5.8 + 20.0662}{0.9} = 28.74\ \text{cm}^{-1}$$

(b) Consider index $n_1 = 3.5$ and a wavelength in vacuum $\lambda_0 = 800$ nm. Here we can work with meters as length units. The frequency spacing for the longitudinal spectrum in the cavity is

$$\Delta f = \frac{c}{2n_1 L} = \frac{300\times 10^{8}}{2\times 3.5\times 600\times 10^{-6}} = 71.429\,\text{GHz}$$

For simplicity, dispersion effects are not considered. The wavelength spacing is given by

$$\Delta\lambda = \frac{\lambda_0^2}{2n_1 L} = \frac{\left(800\times 10^{-9}\right)^2}{2\times 3.5\times 600\times 10^{-6}} = 1.5238\,\text{nm}$$

Problem 3)

For this GaAs/AlGaAs interface, we can use tabulated values and fitting of experimental data (see for instance Appendix A of the book by Chuang)

$$\begin{aligned}
&E_{g1} = 1.424\,\text{eV}\ \ (\text{GaAs})\\
&E_{g2} \approx E_{g1} + 1.247x = 1.7981\,\text{eV}\ \ (\text{Al}_{0.3}\text{Ga}_{0.7}\text{As})\\
&\Delta E_g = 0.3741\,\text{eV}\\
&\Delta E_c \approx 0.67\times\Delta E_g = 250.65\,\text{meV}\\
&\Delta E_c \approx 0.33\times\Delta E_g = 123.45\,\text{meV}
\end{aligned}$$

Reasonable approximate values for the effective density of states are found in the examples of the book by Chuang (Chapter 2) and will suffice here (written for T = 300K)

	GaAs	$\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$
$N_C = 2.51\times 10^{19}\left(\frac{m_e^*}{m_0}\right)^{3/2}$	$N_C = 4.3\times 10^{17}\,\text{cm}^{-3}$	$N_C = 6.94\times 10^{17}\,\text{cm}^{-3}$
$N_V = 2.51\times 10^{19}\left(\frac{m_h^*}{m_0}\right)^{3/2}$	$N_V = 8.87\times 10^{18}\,\text{cm}^{-3}$	$N_V = 1.13\times 10^{19}\,\text{cm}^{-3}$

These results suggest that the *n*-side is slightly degenerate with doping 10^{18} cm^{-3} meaning that the Fermi integral formulation is actually needed to estimate the Fermi level. Also, the assumption of complete ionization may be somewhat questionable but we will still use it. In the p region the elementary semiconductor theory is acceptable, although the Fermi integral formulation is always correct. The two should differ just by several meV in this case. The Fermi integrals can be inverted by programming a simple algorithm, the approximate inverse formulation provided also in the textbook, or an online calculator resource (below, $k_B T = 0.0256$ eV):

***p*-side AlGAAs**

$$p = N_V F_{1/2}\left(\frac{E_V - E_F}{k_B T}\right) \ \Rightarrow\ \frac{N_A}{N_V} \simeq F_{1/2}\left(\frac{E_V - E_F}{k_B T}\right) = 0.0885$$

$$E_F - E_V = -k_B T\times(-2.3935) \approx 61.3\,\text{meV}$$

***n*-side AlGAAs** $$n = N_C F_{1/2}\left(\frac{E_F - E_C}{k_B T}\right) \Rightarrow \frac{N_D}{N_C} \simeq F_{1/2}\left(\frac{E_F - E_C}{k_B T}\right) = 1.4409$$

$$E_F - E_C = k_B T \times 0.86405 \approx 22.12\ \text{meV}$$

Note: the notion of the equilibrium Fermi level actually lying above the conduction band edge in a degenerate semiconductor is questionable from the physical point of view (quasi-Fermi level are “fitting” parameters and can do so). In reality, one needs to consider more carefully the Pauli exclusion principle and include in the model an incomplete ionization analysis (there cannot be more carriers than available states) as well as bandgap narrowing. So, we just need to take the result with a grain of salt as approximate. Essentially, as conductivity rises, we can say that the Fermi level coincides approximately with the band edge, as is usually the case for a metals model. Next, we can find the built in potential as

$$qV_0 = E_g(x = 0.3) + \left(E_F - E_C\right)_n - \left(E_F - E_V\right)_p = 1.7597\ \text{eV}$$

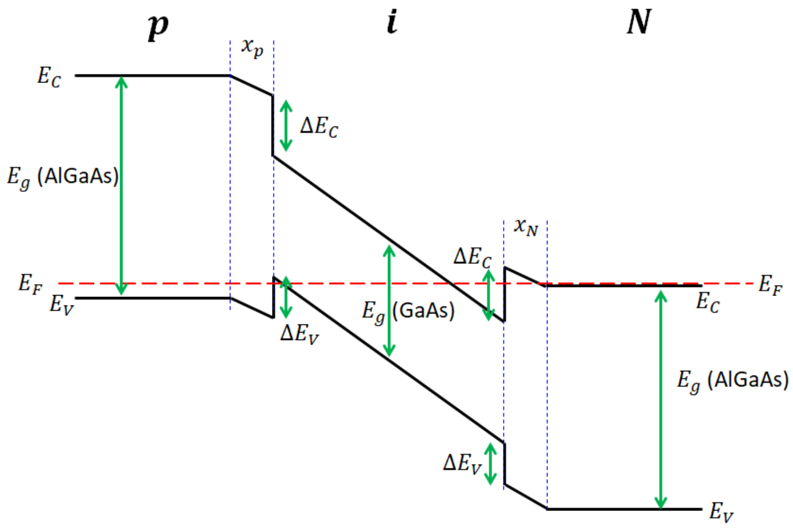
By considering the boundary condition at $x = x_N + W$ and charge neutrality, from the standard *p*-*n* junction model we can write the following relations:

$$V_0 = \frac{qN_A x_p W}{\epsilon_I} + \frac{qN_A}{2\epsilon_p} x_P^2 + \frac{qN_D}{2\epsilon_N} X_N^2 - \frac{qN_D}{2\epsilon_N}\left(x_n + W - (W + x_N)\right)^2$$

$$V_0 = \frac{qN_A x_p W}{\epsilon_I} + \frac{qN_A}{2\epsilon_p} x_P^2 + \frac{qN_D}{2\epsilon_N} X_N^2$$

$$V_0 = \frac{qN_A x_p W}{\epsilon_I} + \frac{qN_A}{2\epsilon_p} x_P^2 + \frac{qN_A^2}{2\epsilon_N N_D} x_P^2$$

from which we obtain $x_p = x_N = 1.274\ \text{nm}$. We have now all the information to draw a rough sketch of the band diagram. Note that in the intrinsic GaAs region there is essentially a linear potential drop.



Problem 4)

(A) FACET OUTPUT Starting from the threshold gain condition

$$\Gamma g_{th} = \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2}$$

we can define the mirror loss from each facet as

$$\alpha_{m,i} = \frac{1}{2L} \ln \frac{1}{R_i}$$

Then, we can directly write the output power from the i-th facet as

$$P_i = \eta_i \frac{\alpha_{m,i}}{\alpha_{m,i} + \alpha_{m,j} + \alpha_i} \frac{\hbar\omega}{q} \left(I - I_{th}\right) = \frac{\ln\left(1/R_i\right)}{\ln\left(1/R_i R_j\right) + 2\alpha_i L} \frac{\hbar\omega}{q} \left(I - I_{th}\right)$$

(B) PLOTS OF FACET POWER We can see the effect of the reflectivities by holding one constant while varying the other, then incrementing the other reflectivity. The result is plotted in Fig. 4.1. Alternatively, we can simultaneously vary the reflectivities of the two mirrors and track the power output from one of them. That result is plotted in Fig. 4.1 as the surface plot. Note that both show that a larger fraction of the power can be collected from the mirror with lower reflectivity, given that the other mirror has a sufficiently high R.

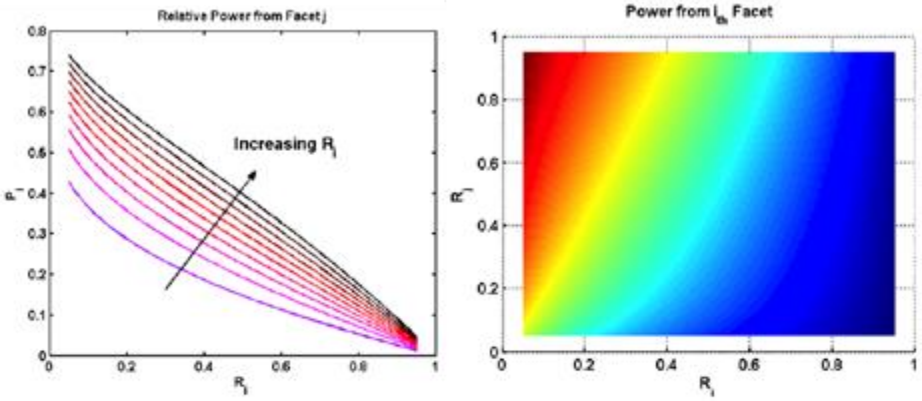


Figure 4.1: Relative output power from facet j as a function of reflectivities i, j. The left is a contour plot for fixed values of R_j whereas the surface plot smoothly varies both reflectivites and monitors the output from the x-axis variation.

(C) FACET COMPARISON If $R_1 = 0.3$ and $R_2 = 0.5$, $P_1 = 0.4156$ and $P_2 = 0.2393$. Therefore, more power comes out of the first mirror than the second, regardless of the injection current I.

Problem 5)

We can use the same formulation as used in Problem 5, except that now the mirrors have the same reflectivity. The figure below shows plots of the emitted power, normalized by current above threshold, as a function of mirror reflectivity and cavity length. The figure summarizes the main trends in laser power emission in terms of $(I - I_{th})$. Of course, further consideration needs to be given to the actual threshold current behavior to judge the quality of the device for a specific purpose. So, the conclusions here are mainly valid for trade-offs in high power laser applications:

- In order to extract more power from the cavity, the mirror reflectivity should be as low as possible, as allowed by the ability of the specific structure to sustain stimulated emission conditions (if too much power leaves the cavity for each round trip, photon population may not remain stable)
- For the same current injected, power emission decreases as the cavity is lengthened. The trade-off here is that a longer cavity allows one to have a bigger injection contact, with higher total current for a given current density. Therefore, with a longer cavity the power density is reduced per unit length at the same injection current, but it may be possible to obtain overall higher power from the structure by increasing the current, although not as efficiently as with a shorter cavity, while easing thermal issues which will have to be considered with current increase.